Shared Private Signal and Information Aggregation

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Introduction

The Rise of the Shared Private Signal

- A signal common to a group of investors but not fully public.
- equity research reports $\rightarrow$ fintech $\rightarrow$ AI

Trade-Off about price informativeness

- **Upside:** Low information acquisiton costs $\Rightarrow$ less uninformed trader, more informed traders
- **Downside:** Correlated Trading Behavior $\Rightarrow$ Amplify the errors from the shared private signal

Research Question:

- How does this signal affect information aggregation?

Price Informativeness

Literature Review

Model and Results

Model Framework

- Based on Grossman & Stiglitz (1980).
- Traders trade a risky asset with unknown payoff.
- Before trading, traders can stay uninformed or acquire one of the following signals:
  - **Independent signal**: each trader observes an i.i.d. signal.
  - **Shared private signal**: traders share the same signal.

Results.

- *If information is cheap:* lowering the information costs can reduce price informativeness.
- *If also* the shared signal is less precise than the independent one, multiple equilibria can arise.

Overview

1. **Introduction**
2. **Trading Stage**
3. **Equilibrium Analysis within trading stage**
4. **Endogenous Information Acquisition**
5. **Conclusion**

Trading Stage - Setup

Assets. Riskless bond (price 1, zero net return) and a *risky* asset with exogenous inelastic supply.
Payoff θ and noisy supply z.

$$\theta \sim \mathcal{N}(\mu_\theta, \tau_\theta^{-1}); \qquad z \sim \mathcal{N}(0, \sigma_z^2).$$

Traders. CARA utility $U(W) = -e^{-\gamma W}$; choose price–contingent demands to maximize expected utility.
Information.

- *AI (A):* $s_A = \theta + \varepsilon_A$, $\varepsilon_A \sim \mathcal{N}(0, \tau_A^{-1})$.
- *Independent (I):* $s_i = \theta + \varepsilon_i$, $\varepsilon_i \sim \mathcal{N}(0, \tau_I^{-1})$ i.i.d..
- *Uninformed (U):* no private signal.

Population shares. $\lambda_A, \lambda_I \geq 0$, $\lambda_U = 1 - \lambda_A - \lambda_I \geq 0$.
$(\theta, \varepsilon_A, \{\varepsilon_i\}_i, z)$ are mutually independent.

Price Informativeness

Conjectured linear price :

$$\begin{aligned} p &= \alpha + a\theta + bs_A - \kappa z \\ &= \alpha + (a+b)\theta + b\varepsilon_A - \kappa z \end{aligned}$$

Coefficients α, a, b, κ are endogenously determined in equilibrium

Signals extracted from price.

A: $\tilde{s}_{p|A} = \dfrac{p - \alpha - bs_A}{a} = \theta - \underbrace{\dfrac{\kappa}{a} z}_{\text{noise from random supply}}, \quad \Rightarrow \tau_p^A = \dfrac{a^2}{\kappa^2 \sigma_z^2}.$

I/U: $\tilde{s}_{p|I} = \dfrac{p - \alpha}{a+b} = \theta + \underbrace{\dfrac{b}{a+b} \varepsilon_A}_{\text{noise from AI signal}} - \underbrace{\dfrac{\kappa}{a+b} z}_{\text{noise from random supply}}, \quad \Rightarrow \underbrace{\tau_p = \dfrac{1}{\mathrm{Var}(\theta \mid p)}}_{\text{price informativeness}} = \dfrac{(a+b)^2}{b^2 \sigma_A^2 + \kappa^2 \sigma_z^2}.$

Equilibrium Condition

Optimal demand: $D(\mathcal{F}) = \frac{\mathbb{E}[\theta \mid \mathcal{F}] - p}{\gamma \operatorname{Var}(\theta \mid \mathcal{F})}$, where $\mathcal{F}$ is the information set available to the trader

Market Clearing Condition:

$$\lambda_A \, D_A(s_A, p) \; + \; \lambda_I \int D_I(s_i, p) di + \; \lambda_U \, D_U(p) \; = \; z.$$

Proposition 1 (Existence and Uniqueness of Linear REE)

For any population shares $\lambda_A, \lambda_I \geq 0$ with $\lambda_A + \lambda_I \leq 1$, there exists a unique linear rational expectations equilibrium (REE). In this equilibrium, the price is given by $p = \alpha + a\theta + bs_A - \kappa z$, where the coefficients (α, a, b, κ) are uniquely determined as functions of the model's exogenous parameters.

Trading Intensities

Trading intensities (hold p fixed):

$$I_A := \lambda_A \frac{\partial D_A}{\partial s_A}\bigg|_p, \qquad I_I := \lambda_I \frac{\partial D_I}{\partial s_i}\bigg|_p.$$

Coefficient–intensity identities:

$$\frac{a}{\kappa} = I_I, \qquad \frac{b}{\kappa} = I_A, \qquad \frac{b}{a} = \frac{I_A}{I_I}.$$

Price-based precisions (in intensities):

$$\tilde{s}_{p|A} = \theta - \underbrace{\frac{1}{I_I}\, z}_{\text{supply noise}} \quad \Rightarrow \quad \tau_p^A = \frac{I_I^2}{\sigma_z^2},$$

$$\tilde{s}_{p|I} = \theta + \underbrace{\frac{I_A}{I_A + I_I}\,\varepsilon_A}_{\text{AI-signal noise}} - \underbrace{\frac{1}{I_A + I_I}\, z}_{\text{supply noise}} \Rightarrow \tau_p = \frac{(I_A + I_I)^2}{I_A^2 \sigma_A^2 + \sigma_z^2}.$$

Price Informativeness

$$\frac{\partial \tau_p}{\partial I_I} > 0, \quad \frac{\partial \tau_p}{\partial \lambda_I} > 0.$$

$$\frac{\partial \tau_p}{\partial I_A} \begin{cases} > 0, & I_A < I_A^\star, \\ = 0, & I_A = I_A^\star, \\ < 0, & I_A > I_A^\star. \end{cases}$$

where $I_A^\star := \dfrac{\sigma_z^2}{I_I\, \sigma_A^2}$.

We can prove that

$$I_A < I_A^\star$$

AI trader tend to reduce their individual demand in order to avoid strategy crowding

$$\frac{\partial I_A}{\partial \lambda_A} > 0, \quad \frac{\partial^2 I_A}{\partial \lambda_A^2} < 0, \quad \frac{\partial}{\partial \lambda_A}\left(\frac{I_A}{\lambda_A}\right) < 0 \quad \Rightarrow \quad \frac{\partial \tau_p}{\partial \lambda_A} = \frac{\partial \tau_p}{\partial I_A}\frac{\partial I_A}{\partial \lambda_A} > 0$$

Endogenous Information Acquisition

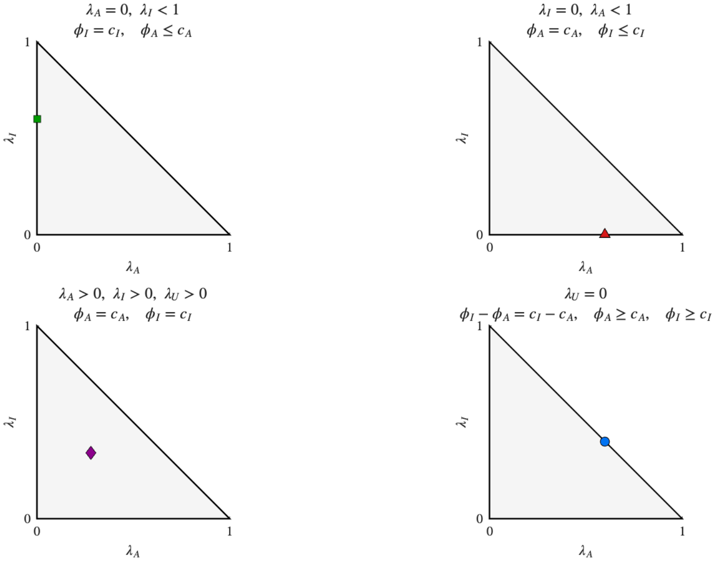
Information costs: Pay $c_A > 0$ for the AI signal; pay $c_I > 0$ for an independent signal.

Information Value: Measure the information advantage over uninformed traders

$$\phi_I(\lambda_A, \lambda_I) = \frac{1}{2\gamma} \ln \left(\frac{\tau_\theta + \tau_I + \tau_p(\lambda_A, \lambda_I)}{\tau_\theta + \tau_p(\lambda_A, \lambda_I)} \right),$$

$$\phi_A(\lambda_A, \lambda_I) = \frac{1}{2\gamma} \ln \left(\frac{\tau_\theta + \tau_A + \tau_p^A(\lambda_A, \lambda_I)}{\tau_\theta + \tau_p(\lambda_A, \lambda_I)} \right).$$

Equilibrium Conditions



$\lambda_U > 0$: Equilibrium Analysis

Proposition 2 (At Most One Equilibrium with $\lambda_U^* > 0$)

For any parameter vector $(\gamma, \sigma_z, \tau_A, \tau_I, \tau_\theta) \in \mathbb{R}_+^5$*, there is at most one information acquisition equilibrium with* $\lambda_U^* > 0$ *for all cost pairs* (c_A, c_I)*.*

Intuition: The value of both information (ϕ_A and ϕ_I) decreases when the share of **either** informed trader share (λ_A or λ_I) rises.

Comparative statics

- $c_A \downarrow \rightarrow \lambda_A \uparrow \rightarrow \tau_p \uparrow \rightarrow \phi_I \downarrow \rightarrow \lambda_I \downarrow \rightarrow \tau_p^A \downarrow, \tau_p$ unchanged.
- $\frac{\partial \lambda_A^*}{\partial \tau_A} > 0, \frac{\partial \lambda_I^*}{\partial \tau_A} < 0 \quad \Rightarrow \tau_A \uparrow \rightarrow \lambda_A \uparrow, \lambda_I \downarrow$
- $\frac{\partial \tau_p^*}{\partial \tau_I} > 0, \text{sign}(\frac{\partial \lambda_I^*}{\partial \tau_I}) = \text{sign}(\tau_\theta + \tau_A - \tau_p). \quad \Rightarrow \tau_I \uparrow \rightarrow$ if τ_I small, then $\lambda_I \uparrow$; otherwise $\lambda_I \downarrow$.

$\lambda_U = 0$: Multiple Equilibria

Proposition 3 (Multiple Equilibria when $\tau_A < \tau_I$)

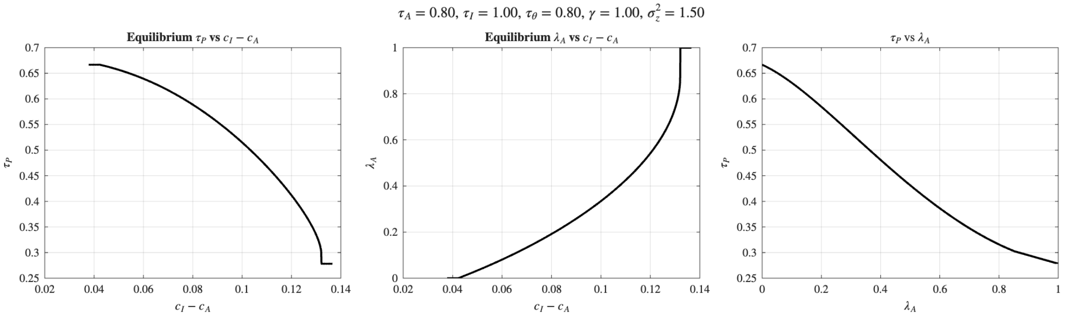
For any parameter vector $(\gamma, \sigma_z, \tau_A, \tau_I, \tau_\theta) \in \mathbb{R}^5_+$ such that $\tau_A < \tau_I$ (AI signal is less precise than independent signal), there exist cost pairs $(c_A, c_I) \in \mathbb{R}^2_+$ for which multiple equilibria with $\lambda_U = 0$ coexist.

Mechanism: Lower c_A $\Rightarrow$ More AI trader (λ_A) $\Rightarrow$ Larger AI signal noise in price $\Rightarrow$ Reduce price informativeness τ_P and higher information value ϕ_I (substitution effect) $\Rightarrow$ If λ_I is low, ϕ_A arises (complementary effect)

Further Implication: Unknown share of informed trader $\Rightarrow$ Multiple equilibria in trading stage & make the market fragile (Banerjee & Green (2015) and Gao, Song & Wang (2013))

$\lambda_U = 0$: Equilibrium Analysis $\tau_A < \tau_I$

When the shared signal is less precise ($\tau_A < \tau_I$), lowering the cost of AI signals (c_A) raises the share of AI traders ($\lambda_A \uparrow$). This shift decreases overall price informativeness (τ_p).



$\lambda_U = 0$: Unique Equilibrium

Theorem 1 (Equilibrium uniqueness when $\tau_A \geq \tau_I$)

For any parameter vector $(\gamma, \sigma_z, \tau_A, \tau_I, \tau_\theta) \in \mathbb{R}^5_+$, *there is at most one information acquisition equilibrium for all cost pairs* (c_A, c_I) *if either of the following conditions holds:*

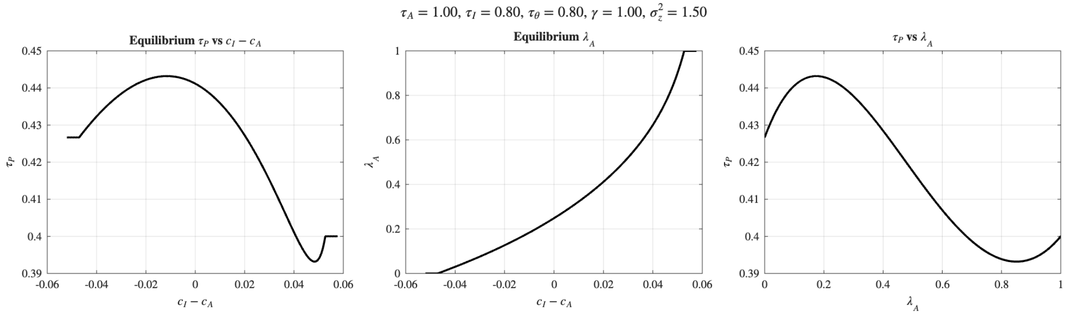
- *AI signal is sufficiently precise:* $\tau_A > \tau_A^* > \tau_I$.
- *AI signal is at least as precise as the independent signal* $(\tau_A \geq \tau_I)$ *and the prior precision is sufficiently low:* $\tau_\theta < \tau_\theta^*$.

Intuition: The complementarity effect still exists, but it is dominated by the substitution effect when the AI signal is more precise.

Numerical Simulation: When $\tau_A = \tau_I$, multiple equilibria were found when $\tau_\theta \approx 5\tau_A$.

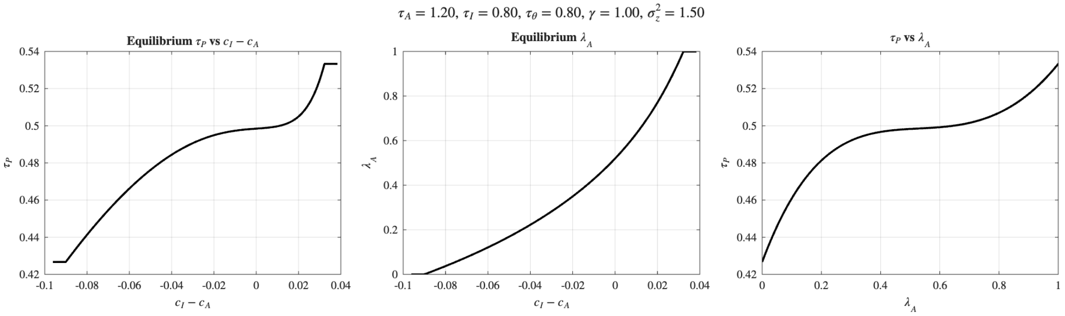
$\lambda_U = 0$: Equilibrium Analysis $\tau_A > \tau_I$

When the shared signal is more precise ($\tau_A > \tau_I$), lowering the cost of independent signals (c_I) reduces the share of AI traders ($\lambda_A \downarrow$). This shift decreases overall price informativeness (τ_p) if the initial equilibrium share of AI traders (λ_A) is either very low or very high.



$\lambda_U = 0$: Equilibrium Analysis $\tau_A \gg \tau_I$

When the shared signal is precise enough ($\tau_A \gg \tau_I$), lowering the cost of independent signals (c_I) reduces the share of AI traders ($\lambda_A \downarrow$). This shift decreases overall price informativeness (τ_p).



Relation to the Literature

1. **Information Aggregation in Financial Market**
   - Lou et al. (2019), Parsa and Ray (2017), Figlewski (1982), Goldstein and Yang (2015).
2. **Endogenous Information Choice**
   - Veldkamp (2006), Hellwig and Veldkamp (2009), Chahrour (2014).
3. **Publiic Information and Private information**
   - Morris and Shin (2002), Angeletos and Werning (2006).
4. **Price Informativeness and Information Acquisition Cost**
   - Grossman & Stiglitz (1980): Lower information costs improve price informativeness.
   - Recent empirical studies report contradictory findings (e.g., Sammon, 2025; McClure et al., 2023).

Conclusion and Future Work

Results

- The trade-off of shared private signal.
- Price informativeness effects are complex and depend on the trader shares and relative precision.
- Regulator should note the potential risks when information costs are low and shared private signals are relatively imprecise.

Future Plan

- Acquiring both signals. (done)
- Identical signal $\rightarrow$ Correlated signal. (done)
- Information sale.

Price Informativeness

- **Capital allocation:** More informative prices direct savings toward high-value projects and away from lemons.
- **Corporate decisions:** Managers, boards, and investors learn from prices when issuing, investing, or doing M&A.
- **Market quality:** Better price discovery reduces adverse selection, supports liquidity, and lowers the cost of capital.
- **Policy & stability:** Guides disclosure and trading rules; low informativeness or correlated errors can amplify mispricing and fragility.

Rational-Expectations Equilibrium (REE)

- **Core idea:** Agents form expectations using all available information — including market prices.
- **Correct beliefs:** In equilibrium, those expectations are correct given the way prices are generated.
- **Equilibrium outcome as signals:** Prices aggregate dispersed private information; agents understand this and learn from prices.
- **Fixed point:** Market-clearing prices and agents' beliefs are mutually consistent.